Diffraction Analysis of an Ellipsoidal Wave Front

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The study of Diabetic Retinopathy Sulla Swall Dial Causes of Diabetic Retinopathy : Types of diabetic retinopathy : Early diabetic retinopathy Advanced diabetic retinopathy Risk factors for diabetic retinopathy : Prevention of diabetic retinopathy Diagnosis of diabetic retinopathy : Symptoms of diabetic retinopathy : Tests for diabetic retinopathy : Complications of diabetic retinopathy : Treatment of diabetic retinopathy : Early diabetic retinopathy : Managing diabetes : Treatment of advanced diabetic retinopathy : Eye Injections (Anti - VEGF drugs) : Laser treatment of diabetic retinopathy : Eye surgery (vitrectomy) :

الملخص:

تم اشتقاق تعبير عن الحيود من المبادئ الأولى عند واجهة موجة إهليلجية.يظهر حيود واجهات الموجات الكروية والمستوية كحالات محددة من نفس الشيء. تبين أن مناطق نصف فترة فريسنل من واجهة الموجة الإهليلجية تعتمد على الإهليلجية لواجهة الموجة.

Abstract: An expression for diffraction from first principles at an ellipsoidal wavefront is derived. The diffraction of spherical and planar wavefronts are shown as specific cases from the same. The areas of Fresnel half period zones from the ellipsoidal wavefront are shown to be dependant on the ellipticity of the wavefront.

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1-INTRODUCTION

We are aware that when light waves pass through an aperture or are partly blocked by obstacles. they spread beyond the limit of the geometric shadow of the aperture or obstacle by the phenomenon known as diffraction. to Huygen's principle, points on a wavefront are considered as new point sources which in their turn generate a spherical wavefront. Max Born and Emil Wolf ,[21 analysed the same phenomenon by a more sophisticated wave theory due to Fresnel. Cons-truction of Fresnel half period zones due to cylindrical wavefront and the consequent theory of Cornu's spiral has also been adequately dealt in standard texts [3][4,5] on Optics.However diffraction from an ellipsoidal wave front, although a more general case, does not seem to have been worked out by the earlier workers.Further diffraction phenomenon has been of central importance with the advent of lasers, holographic imaging and holographic optics.Hence the authors have made an attempt in this paper i) to derive an expression for diffraction of an ellipsoidal wavefront from first principles and it) to treat the diffraction of spherical and planar wavefronts as particular cases from the same analysis. This analysis embodies the basic principles involved in the phenomenon of diffraction and elegantly brings out the importance of the ellipticity in the control of the intensity produced at the point of observation due to Fresnel half period zones of the wavefront intercepted by the aperture or the obstacle.

2. ANALYSIS OF DIFFRACTION OF AN ELLIPSOIDAL WAVEFRONT.

2.1. DETERMINATION OF AN ELLIPTICAL CAP AREAOF AN ELLIPSOIDAL WAVEFRONT AND ITS ANA-LOGY WITH THE CIRCULAR CASE.

The conventional theory Ti of Diffraction from a spherical wavefront is based on the principle of determining the areas of spherical caps consisting of L and (L-1) half period zones and thence arrive at the area of the Lth. zone for determining the intensity at any point on the axial line of the wavefront. The same method is adopted in the treatment of diffraction from an ellipsoidal wavefront.The area of the shaded ring element at any point (x, y) on the ellipsoidal wavefront from Fig. 1 can be set as

$$S = 2\pi y\ \ ds = 2\pi y\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}} dx$$

The ellipsoidal cap area between the limits a and x' may now be written as

$$S_{cap} = \int_a^{x'} 2\pi y\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{2}} dx \qquad (1)$$

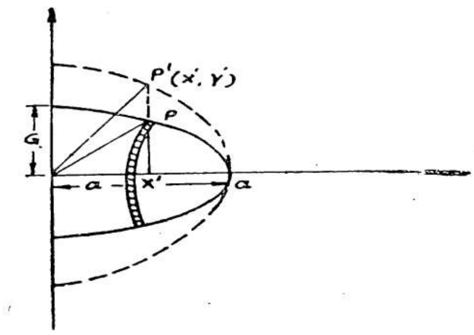

Fig. 1

The Ellipsoidal Cap Areas Covering L and L-1 Zones. (The Shaded Element Gives the Area of the Lth Zone).

Substituting $x = \left(\frac{a}{e}\right) sin\varphi$. and using the fact that $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = I$, where $e$is the eccentricity of the ellipse, we have

$$S_{cap} = \int_0^{\varphi'} \frac{1}{e}(e^2 - sin^2\varphi)^{\frac{1}{2}}\left\{1 + \frac{b^2 sin^2\varphi}{a^2(e^2 - sin^2\varphi)}\right\}^{\frac{1}{2}} \times \frac{a}{e} cos\varphi\, d\varphi$$

....(2)

Here

$\varphi_o = sin^{-1}e \ and \ \varphi' = sin^{-1}(\frac{x'e}{a})$

Invoking $b^2 = (1 - e^2)a^2$ equation (2)may be simplified as

$$S_{cap} = \frac{2\pi ab}{e}\int_a^{x'} cos^2\varphi \ d\varphi$$

Integranting and expressing the same in terms of the original variables, we have

$$S_{cap} = \frac{\pi ab}{e}\left[sin^{-1}\frac{xe}{a} + \frac{xe}{a}\sqrt{1 - \left(\frac{xe}{a}\right)^2}\right]_a^{x'} \quad (3)$$

We know that as e -> 0, lim. (1/e) $sin^{-1}(ex/a) = x/a$

Hence equation (3) can be shown to be,

$$S_{cap} = -2\pi a^2\left(1 - \frac{x'}{a}\right) = -2\pi a^2(1 - cos\varphi') \quad (4)$$

Equation (4) above represents the area of a spherical cap of a radius between the planes x= a and x=x'

2.2. FORMULATION OF FRESNEL HALE PERIOD ZONES FROM THE ELLIPSOIDAL WAVEFRONT.

Let us define the area of the L_{th} zone, which is the strip of the ellipsoid between the planes $x = x_L$ and $x = x_{L-1}$as

$$A_{Lth} = S_L - S_{L-1} \quad (5)$$

whereS_L , and S_{L-1} are respectively the cap areas consisting of L and L_ 1 Fresnel half period zones. using equation (3), we obtain

$$A_{Lth} = \frac{\pi a^2}{e}\sqrt{1 - e^1}\left[sin^{-1}p + p\sqrt{1 - p^2}\right]_{P_{L-1}}^{P_L} (6)$$

Where $P_k = (\frac{ex_k}{a})$

In order to understand the effect of the ellipsoidal nature of the wavefront, we derive below an asymptotic expression for A_L for small eccentricities e. Let us defineS_L , as

$$S_L = \frac{\pi a^2}{e}(1-e^2)^{\frac{1}{2}}\left[sin^{-1}P_L + P_L\sqrt{1-P_L{}^2}\right] \quad (7)$$

Using the expansions of the terms within brackets up to terms of the order es equation (7) can be written as

$$S_L = \pi a^2\left(\frac{1}{e}-\frac{1}{2}e\right)\left[\left(P_L+\frac{1}{6}P_L{}^3\right)+\left(P_L-\frac{1}{2}P_L{}^3\right)\right]+O(e^4)$$
(8)

where the term $O(e^4)$refers to the neglected of ordere^4 . Substituting for P_L and simplifying equation (8) we have

$$S_L = \pi a^2\left[\frac{2x_l}{a}-\left\{\frac{x_L}{a}+\frac{1}{3}\left(\frac{x_L}{a}\right)^3\right\}e^2\right]+O(e^4) \quad (9)$$

A similar expression for St-, can be got by replacingx_L by x_{L-1} in equation (8). Now the

area of the L_{th}zone can be expressed as

$$A_{Lth} = \pi a^2\left[\frac{2x}{a}-\left\{\frac{x}{a}+\frac{1}{3}\left(\frac{x}{a}\right)^3\right\}e^2\right]_{x_{L-1}}^{x_L}+O(e^4) \quad (10)$$

In deriving the asymptotic expression for S, in terms of r_L , $r_o$and e we use from Fig. (2

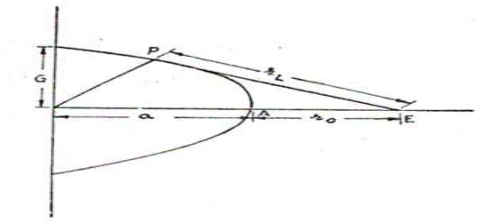

Fig. 2

The section of an Ellipsoidal Wavefront showing r_o,$r_L$and E, the Point of Observation.

r_o=the distance of E, the point of observation from the apex of the ellipse along its axis.

$r_L{}^2 = Y_L{}^2 + (r_o + a - x_L)^2$(11)

where $r_L = r_o + \left(\frac{L^\lambda}{L}/2\right)$ for the L_{th} zone.

Replacing y_L from the equation of the ellipse, $(x_L{}^2/a^2) + (y_L{}^2/b^2)$andusing $b^2 = (1 - e^2)a^2$,

we have a quadratic equation forx_L given by

$e^2 x_L^2$-$2(r_o + a)x_L + (r_o + a)^2 + a^2 - r_L{}^2 - a^2 e^2$=0(12)

Solving equation (12) for x_L, we have

$x_L = \frac{1}{e^2}\left[(r_o + a) \pm \{(r_o + a)^2 - e^2\{(r_o + a)^2 + e^2 - r_L{}^2\} - a^2 e^4\}^{\frac{1}{2}}\right]$
(13)

Of the two roots for XL, the root with the + ve sign is not acceptable as it makes$x_L > (r_o + a)$ Hence we take the root with the -ve sign. Expanding the same up to terms of e^2. we have,

$x_L = \frac{1}{2}(r_o + a)R_L^2 + \left\{\frac{1}{8}(r_o + a)R_L^2 - \frac{a^2}{2(r_o+a)}\right\}e^2 + O(e^4)$
(14)

Where

$R_L^2 = \frac{(r_o+a)^2+a^2-r_L^2}{(r_o+a)^2}$ (15)

Now substituting from equations (9) and (14)

In equations (5) and simplifying we obtain

$$A_{Lth} = \pi a^2 \left(1 + \frac{r_o}{a}\right)(R_L^2 - R_{L-1}^2)\Big[1 + \left\{-\frac{1}{2} + \frac{1}{4}(R_L^2 + R_{L-1}^2) - \frac{1}{24}\left(1 + \frac{r_o}{a}\right)^2 (R_L^4 + R_L^2 R_{L-1}^2 + R_{L-1}^4)\right\} e^2\Big]$$
(16)

3. RESULTS AND DISCUSSION

Equation (14) has two terms. The first term does not contain e and the second term contains e^2.Substituting for R from equation (13), taking r as (r_o + $L^{\lambda}/2$) and simplifying the first term of equation (16), we have the case of a circular wavefront given as

$$A_{Lth}^s = \pi a^2 \left(1 + \frac{r_o}{a}\right)(R_L^2 - R_L^2) = -\frac{\pi a r_o \lambda}{a + r_o} \quad (17)$$

Here $A_{Lth}{}^s$ refers to the area of the L_{th} spherical zone. The -ve sign can be set as $(-1)^{m+1}$ in order to incarporate the effect of odd and even Fresnel zones, i.e. when m is odd Asth is positive and when m is even Astn is negative. This shows that when the aperture intercepts odd number of zones for a, point of observation Eat a distancer. from the apex of the wavefront the point is bright, where as in the other case odd and even zones annul the effect of each other and the point is dark.

If however the source is drawn to infinity equation (17) leads to the case of a planar wavefront and we have

$A_{Lth}(planar) = (-1)^{m+1}\pi\lambda r^o$(18)

One can write equation (16) as

$A_{Lth} = A_{Lth}^S(1 - \varepsilon_{th}e^2) + O(e^4)$ (19)

where ern is the coefficient of e? in equation (16).This is brought as the correction term to thespherical term due to the ellipsoidal nature of the wavefront. It may be shown that en is always positive. Hence it is seen that the ellipsoidal nature of the wavefront reduces the area of the zones as we proceed from the centre of the zone plate.

5. Conclusions

1) The analysis of the zone plate construction for an ellipsoidal wavefront from first principles presented in this paper is a more general case and the diffraction of circular and planar wavefronts can be drawn as particular cases of this analysis.

2) It is also clear that although ideal point sources or ideal spherical wavefronts are not

realised in practice, for small ellipticities, (i.e. departures from spherical nature of the wavefront) the effect at the point of observation is that due to a spherical wavefront only. This is because of the e3 term involved in equation (16), Thus the analysis enables one to confirm that Huygen's hypothesis of point sources and generation of spherical wavefronts from them is valid.

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